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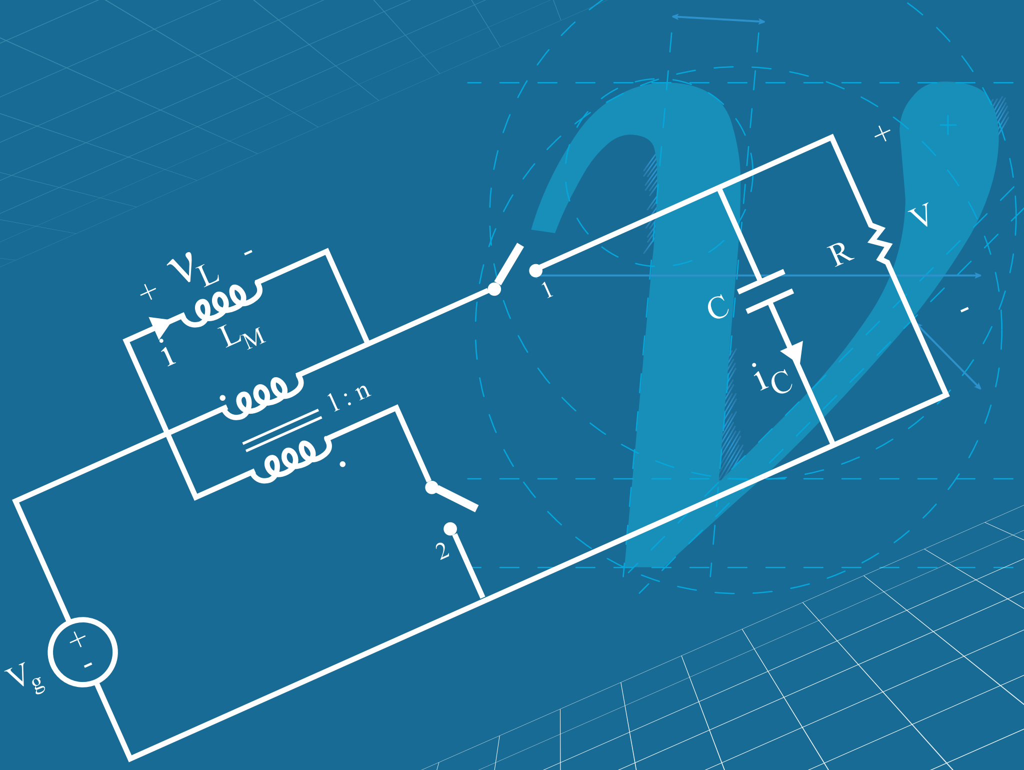

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HYDROMETER CALIBRATION: LINKING SIM NMIS TO THE EURAMET KEY COMPARISON REFERENCE VALUE OF EURAMET.M.D-K4

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Resumen

Un elemento importante para soportar las Capacidades de Medición y Calibración (CMCs) de los Institutos Nacionales de Metrología (INM) es su participación en comparaciones internacionales, y mediante ello analizar el grado de equivalencia entre sus resultados de medición contra los correspondientes valores de referencia. En el presente artículo se describe un método para ligar los resultados de la comparación clave del SIM con valor de referencia de la comparación clave de EURAMET en la calibración de hidrómetros de alta exactitud.

Palabras clave: Metrología, comparaciones clave, liga de comparaciones, calibración de hidrómetros.

Abstract

An important tool that supports the Calibration and Measurement Capabilities (CMCs) of National Metrology Institutes (NMI) is their participation in international comparisons, and thereby to analyze the degree of equivalence between their measurement results against the corresponding reference values. This paper describes a method for linking the results of the SIM key comparisonwith the EURAMET key comparison reference value on calibration of high accuracy hydrometers.

Keywords: Metrology, key comparisons, link of comparisons, hydrometers calibration.

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1. INTRODUCTION

Calibration of hydrometers is an important activity of metrology due to importance of density in quality of products, (sugar concentration, percentage of alcohol per volume, etc.) as well as part of measurements for quantity (flow and volume), among others.

Due to the above reasons it is important for laboratories to demonstrate their capability in density meter calibration. A possibility is to participate in comparisons and knowing the degree of equivalence against to the corresponding reference values.

Within the EURAMET[1] region a key comparison on hydrometer calibration was carried out from October 2003 to May 2004 under the identification of EURAMET.M.D-K4. Another key comparison between SIM[2] NMIs on hydrometer calibration was carried out from April 2007 to October 2008 identified as SIM.M.D-K4. INRIM-Italy and CENAM-Mexico were the pilot laboratories for the two key comparisons respectively.

With the intention of linking these two key comparisons, a bilateral comparison between CENAM-Mexico and INRIM-Italy was carried out from June to October 2007; this bilateral comparison was identified as SIM.M.D-S1.

In the present paper a methodology is presented for linking the results of the participant NMIs of the SIM.M.D-K4 to the key comparison reference value of EURAMET.M.D-K4 ($KCRV_{EU}$), through the bilateral comparison SIM.M.D-S1.

2. PARTICIPANTS AND TRAVELLING STANDARDS OF COMPARISONS

For this study three comparisons were required, two of them were key comparisons organized within two different metrological organizations, and the last was a linking comparison (bilateral comparison) organized between CENAM and INRIM belonging to SIM and EURAMET respectively.

2.1 EURAMET.M.D-K4

This key comparison was organized within Europe region with the participation of the following NMIs:

- IMGC - Italy(now INRIM)
- OMH - Hungary
- PTB - Germany
- BNM- LNE - France
- IPQ - Portugal
- MIKES - Finland
- BEV - Austria
- UME - Turkey
- GUM - Poland
- SMU - Slovakia
- VNIIM - Russia

The travelling standards used for the comparison were two sets of high accuracy hydrometers within the following ranges and scale divisions:

Figure 1. Hydrometers of high accuracy.
Source: CENAM

Table 1. Set 1 for EURAMET.M.D-K4

Range	Scale division
0.6100 g/cm^3 - 0.6200 g/cm^3	0.0001 g/cm^3
0.8100 g/cm^3 - 0.8200 g/cm^3	0.0001 g/cm^3
1.0000 g/cm3 -1.0100 g/cm3	0.0001 g/cm^3
1.2900 g/cm3 -1.3000 g/cm3	0.0001 g/cm^3

Source: Authors

Table 2. Set 2 for EURAMET.M.D-K4

Range	Scale division
0.6000 g/cm^3 - 0.6100 g/cm^3	0.0001 g/cm^3
0.8000 g/cm^3 - 0.8100 g/cm^3	0.0001 g/cm^3
0.9900 g/cm3 -1.0000 g/cm3	0.0001 g/cm^3
1.2900 g/cm3 -1.3000 g/cm3	0.0001 g/cm^3

Source: Authors

2.2 SIM.M.D-K4

The NMIs from America region that took part in the SIM.M.D-K4 were the following:

- CENAM - Mexico
- BSJ - Jamaica
- CENAMEP - Panama
- CESMEC - Chile
- IBMETRO - Bolivia
- INDECOPI - Peru
- INEN - Ecuador
- INMETRO - Brazil
- INTI - Argentina
- LACOMET - Costa Rica
- LATU - Uruguay
- NIST - United States
- NRC - Canada
- SIC - Colombia

The travelling standards use for the comparison were two similar sets of hydrometers within the following ranges and scale divisions:

Table 3. Set for SIM.M.D-K4

Range	Scale division
0.6100 g/cm^3 - 0.6200 g/cm^3	0.0001 g/cm^3
0.8000 g/cm^3 - 0.8100 g/cm^3	0.0001 g/cm^3
0.9900 g/cm3 -1.0000 g/cm3	0.0001 g/cm^3
1.2900 g/cm3 -1.3000 g/cm3	0.0001 g/cm^3

Source: Authors

2.3 SIM.M.D-S1

In order to link the above comparisons, a bilateral comparison was organized between CENAM and INRIM. CENAM acted as pilot laboratory in the SIM key comparison, and INRIM was the pilot laboratory for the EURAMET key comparison. This bilateral comparison was registered as SIM supplementary comparison with the code SIM.M.D-S1.

Participants of the SIM.M.D-S1

- INRIM - Italy
- CENAM - Mexico

The travelling standards used for the SIM.M.D-S1 were two hydrometers with the following characteristics:

Table 4. Travelling standars for the SIM.M.D-S1

Range	Scale division
0.8000 g/cm^3 - 0.8200 g/cm^3	0.0002 g/cm^3
1.1800 g/cm^3 - 1.2000 g/cm^3	0.0002 g/cm^3

Source: Authors

3. ANALYSIS OF RESULTS OF THE THREE COMPARISONS

From the results of participants of EURAMET.M.D-K4, a $KCRV_{EU}$ for each nominal value was calculated. These $KCRV_{EU}$ were calculated either as the weighted mean or as the median from results of participants, according to Cox (2002).

The degree of equivalence is calculated as the difference between two values, usually the result reported by a participant and the reference value of a particular key comparison where the laboratory took part. This difference is evaluated joint with the uncertainty of such difference.

Degrees of equivalence *DoEs* between the values reported by participants and the corresponding $KCRV_{EU}$, $D_{i.(EURAMET)}$ were calculated as well as the degrees of equivalence *doEs* for each pair of participants of EURAMET.M.D-K4 (Lorefice *et al*, 2008).

As concerns SIM.M.D-K4 a similar analysis was done *Becerra* (2009). A $KCRV_{SIM}$ for each nominal value was calculated as the weighted mean of results of participants.

DoEs between participating NMIs and the $KCRV_{SIM}$ were calculated as well as *doEs* among participants of SIM.M.D-K4.

Finally, in the analysis of SIM.M.D-S1 (Becerra & Lorefice, 2009) as this was a bilateral comparison there was not need to calculate *RV* but *doEs* between both participants were calculated.

In order to link the results of the SIM NMIs to the *KCRV* of EURAMET, the following analysis was done:

- *DoEs* between results reported by INRIM and the $KCRV_{EU}$ in EURAMET.M.D-K4,

$$D_{INRIM} = INRIM_{EU} - KCRV_{EU} \tag{1}$$

- *doEs* between results reported by NMI *i* and CENAM in SIM.M.D-K4,

$$d_{i(SIM)} = X_{i(SIM)} - CENAM_{SIM} \tag{2}$$

- *doEs* between results reported by CENAM and INRIM in SIM.M.D-S1

$$d_{BIL} = CENAM_{BIL} - INRIM_{BIL} \tag{3}$$

in order to calculate the *DoEs* between the SIM NMI *i* and the $KCRV_{EU}$ we have to calculated as follow:

$$D_i' = d_{i(SIM)} + d_{BIL} + D_{INRIM} \tag{4}$$

D'_i *DoEs* between results reported by SIM NMI*i* and the $KCRV_{EU}$

$X_{i(SIM)}$ Value reported by SIM NMI*i*

In order to calculate (4), it is important to assume that results reported by CENAM in both comparison SIM.M.D-K4 and SIM.M.D-S1 are strongly correlated. The same is assumed for INRIM, results reported by INRIM in both comparisons EURAMET.M.D-K4 and SIM.M.D-S1 are strongly correlated too.

The nominal values measured in the different comparisons are not the same, and due to this reason it is not possible to calculate (4) for each individual value.

The linking procedure used is described in Lorefice, Becerra y Pennecchi (2009), it consists in calculating a regression curve for the corresponding DoEs. According to the specific purpose of this work straight lines (first order curves) were calculated for D_{INRIM}, d_{BIL}, $d_{i(SIM)}$.

The straight line function is the first option for the fitting curves, where *DoEs* are considered proportional to the nominal density.

In order to introduce all set of values a matrix equation takes the form,

$$\mathbf{D} = \boldsymbol{\rho}\boldsymbol{\beta} - \boldsymbol{\varepsilon} \tag{5}$$

where **D** is the column vector of degrees of equivalence, **ρ** is the matrix of nominal values of density, **ε** is the column vector of measurement errors and **β** is the column vector which contains the slope *m* and the intercept *b* of the proposed function, a straight line,

$$\mathbf{D} = \begin{bmatrix} D_1 \\ D_2 \\ \vdots \\ D_{n-1} \\ D_n \end{bmatrix},\ \boldsymbol{\rho} = \begin{bmatrix} \rho_1 & 1 \\ \rho_2 & 1 \\ \vdots & \vdots \\ \rho_{n-1} & 1 \\ \rho_n & 1 \end{bmatrix},\ \boldsymbol{\beta} = \begin{bmatrix} m \\ b \end{bmatrix},\ \boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_{n-1} \\ \varepsilon_n \end{bmatrix}$$

The weighted least squares method (WLS) was used to calculate the regression curves for the corresponding *DoEs*.

The solution of (5) by WLS is (Lorefice, Becerra y Pennecchi, 2009)

$$\hat{\boldsymbol{\beta}} = \left(\boldsymbol{\rho}^{\mathrm{T}}\boldsymbol{\psi}_{\mathbf{D}}^{-1}\boldsymbol{\rho}\right)^{-1}\boldsymbol{\rho}^{\mathrm{T}}\boldsymbol{\psi}_{\mathbf{D}}^{-1}\mathbf{D} \tag{6}$$

The weighing matrices $\boldsymbol{\Psi}_{\mathbf{D}}^{-1}$, were formed by the variance (and covariance) of the corresponding *DoEs* (Lorefice, Becerra, Pennecchi, s.f; JCGM 100, 2008).

Variances were calculated as the square of the uncertainties of *DoEs* reported in each comparison, and the covariances were calculated with pairs of the uncertainties of *DoEs* and the corresponding correlation coefficient.

For the correlation coefficient it was assumed 0.9 for *DoEs* corresponding to the calibration of same hydrometer and 0.3 for *DoEs* corresponding to the calibration of different hydrometers. These correlation coefficients were assumed based on the experience of the authors.

The calculated curves take the following form,

$$D(\rho_i) = m\rho_i + b - \varepsilon_i \quad (7)$$

Where:

$D(\rho_i)_i$ *DoEs* calculated for the corresponding comparison in g/cm³
m slope of the straight line calculated (dimensionless)
b intercept of the straight line calculated in g/cm³
ε_i adjustment error for the proposed function in g/cm³

In order to check the agreement between predicted and observed values the chi-square test χ^2 was used.

Table 5. Fitting coefficients for the continuous functions $d_{i(SIM)}(\rho)$

$d_{i(SIM)}(\rho)$	b'', g/cm³,	m''
$d_{SIC(SIM)}(\rho)$	1.539 x 10-05	-1.921 x 10-05
$d_{LATU(SIM)}(\rho)$	6.071 x 10-05	-7.332 x 10-05
$d_{INDECOPI(SIM)}(\rho)$	6.463 x 10-05	-4.714 x 10-05
$d_{INTI(SIM)}(\rho)$	4.655 x 10-05	-9.329 x 10-05
$d_{NIST(SIM)}(\rho)$	1.463 x 10-05	-8.212 x 10-06
$d_{NRC(SIM)}(\rho)$	1.070 x 10-04	-1.486 x 10-04
$d_{CENAMEP(SIM)}(\rho)$	1.397 x 10-04	-9.877 x 10-05

Source: Authors

3.1 Calculated continuous functions of *DoEs*

The calculated curve coefficients for $D_{INRIM}(\rho)$ in the dominium from 0.6005 g/cm³ to 1.2995 g/cm³ is:

b = 5.81029 x 10-06 g/cm³
m = -1.84023 x 10-05

The calculated curve coefficients for $d_{BIL}(\rho)$ in the dominium³ from 0.802 g/cm³ to 1.198 g/cm³ is:

b'= -2.31066 x 10-05 g/cm³
m'= 1.71067 x 10-05

Both approximations were checked by χ^2 test.

For SIM NMIs the following coefficients for curves to describe $d_{i(SIM)}(\rho)$ were calculated in the dominium from 0.601 g/cm³ to 1.299 g/cm³, except for $d_{NRC(SIM)}$ which is the dominium from 0.801 g/cm³ to 1.299 g/cm³ because NRC did not participate in the lower range in the SIM comparison.

$d_{i(SIM)}(\rho)$ describes the difference (2) as a continuous function.

The other six participants of SIM.M.D-K4, did not pass the χ^2 test for the proposed function.

3.2 Calculation of *DoEs* of SIM NMIs respect to the $KCRV_{EU}$

The continuous functions of *DoEs* of SIM NMIs respect to $KCRV_{EU}$, as continuous functions $D'_i(\rho)$, were calculated by the following formula (*Lorefice, Becerra y Pennecchi, 2009*):

$$D'_i(\rho) = d_{i(SIM)}(\rho) + d_{BIL}(\rho) + D_{INRIM}(\rho) \quad (8)$$

This takes the following form:

$$D'_i(\rho) = (m''+m'+m)\rho_i + (b''+b'+b) - \varepsilon'_i \quad (9a)$$

or

$$D'_i(\rho) = (m^*)\rho_i + (b^*) - \varepsilon'_i \quad (9b)$$

where ε'_i is the adjustment error for the resulting function in g/cm³, $m^* = m'' + m' + m$ and $b^* = b'' + b' + b$.

For CENAM's *DoEs* with respect to $KCRV_{EU}$, $D'_{CENAM}(\rho)$, the continuous function was calculated for the following formula:

$$D'_{CENAM}(\rho) = d_{BIL}(\rho) + D_{INRIM}(\rho) \quad (10)$$

or presented as the addition of the coefficients:

$$D'_{CENAM}(\rho) = (m'+m)\rho_i + (b'+b) - \varepsilon_i \quad (11)$$

The calculated coefficients for the continuous functions of $D'_i(\rho)$ are listed in Table 6.

The dominium $D'_i(\rho)$ of is from 0.802 g/cm³ to 1.198 g/cm³, due to the fact that the linking comparison SIM.M.D-S1 is within the same dominium. The calculated $D'_i(\rho)$ are shown in Fig. 2.

Table 6. Fitting coefficients for the continuous functions $D'_i(\rho)$

$D'_i(\rho)$	b^*, g/cm³,	m^*
$D'_{CENAM}(\rho)$	-1.730 x 10^{-05}	-1.296 x 10^{-06}
$D'_{SIC}(\rho)$	-1.911 x 10^{-06}	-2.051 x 10^{-05}
$D'_{LATU}(\rho)$	4.341 x 10^{-05}	-7.462 x 10^{-05}
$D'_{INDECOPI}(\rho)$	4.733 x 10^{-05}	-4.844 x 10^{-05}
$D'_{INTI}(\rho)$	2.925 x 10^{-05}	-9.458 x 10^{-05}
$D'_{NIST}(\rho)$	-2.664 x 10^{-06}	-9.507 x 10^{-06}
$D'_{NRC}(\rho)$	8.971 x 10^{-05}	-1.499 x 10^{-04}
$D'_{NRC}(\rho)$	1.224 x 10^{-04}	-1.000 x 10^{-04}

Source: Authors

3.3 Uncertainty of *DoEs* of SIM NMIs respect to the $KCRV_{EU}$

In order to calculate the uncertainty of $D'_i(\rho)$, numerical simulations by Monte Carlo method were done for each $D'_i(\rho)$ (JCGM 100, 2008; JCGM 101, 2008), see Fig. 3.

For the numerical simulations, *DoEs* and their corresponding standard uncertainties were considered as the mean value and the standard deviation of normal probability density distributions of the input quantities. For the numerical simulation 100 000 trials were done for each $D'_i(\rho)$.

The normalized errors to an approximated 95% of confidence level were calculated with the following expression:

$$E_n = \frac{D'_i(\rho)}{U(D'_i(\rho))} \quad (12)$$

In Fig. 4 are shown the curves of the normalized errors of $D'_i(\rho)$, the accepted values are ≤ 1.

In Table 7 are shown the values of $D'_i(\rho)$, $U(D'_i(\rho))$, and E_n calculated for selected densities within the dominium of the functions.

5. CONCLUSIONS

In the present paper were presented the *DoEs* of SIM NMIs to the $KCRV_{EU}$, $D'_i(\rho)$.

From fourteen participants of SIM.M.D-K4, only eight participants could be linked to the *KCRV* of the EURAMET.M.D-K4. The other six participants failed the χ^2 test for the proposed function.

All SIMNMIs that passed the χ^2 test had normalized errors smaller than 1 when were contrasted against the $KCRV_{EU}$,

The straight line function is the first option for the fitting curves, where *DoEs* are considered proportional to the nominal density.

Indeed it is possible to calculated other functions that could fit better than straight line (e.g. by a second order polynomial function), but the straight line is one of the simplest assumption.

In the straight line function, the systematic effects could be divided two groups, the constant effects which could be included in the intercept (*b*) and the relative effects which could be included in the slope (*m*), see formula (7).

The uncertainty of the $D'_i(\rho)$ were dominated by the uncertainties of the doEs of the linking comparison SIM.M.D-S1 due to the scale division of the travelling standards used in such bilateral comparison.

A similar analysis could be done for linking SIM.M.D-K4 to the CCM key comparison CCM.D-K4 which will be started in 2010, activity waiting to be done, once results from CCM.D-K4 be available.

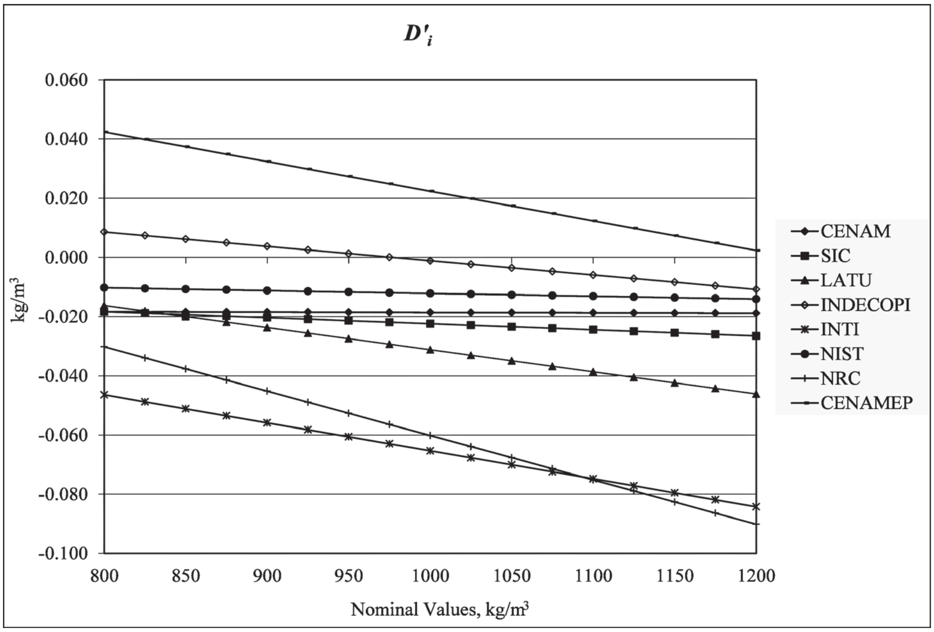

Figure 2. *DoEs* between SIM NMIs and $KCRV_{EU}$, $D'_i(\rho)$.

Source: Authors

Table 7. Calculated values of *DoEs* of SIM NMIs regards to the $KCRV_{EU}$, their expanded uncertainties associated and the calculated normalized errors, for selected values of density within the dominium of the functions

ρ	D'CENAM	U(95%)	En	D'SIC	U(95%)	En	D'LATU	U(95%)	En	D'INDECOPI	U(95%)	En
g/cm³	g/cm³	g/cm³		g/cm³	g/cm³		g/cm³	g/cm³		g/cm³	g/cm³	
0.80	-1.83 x 10^{-05}	5.52 x 10^{-05}	-0.33	-1.83 x 10^{-05}	6.40 x 10^{-05}	-0.29	-1.63 x 10^{-05}	5.67 x 10^{-05}	-0.29	8.58 x 10^{-05}	6.73 x 10^{-05}	0.13
0.83	-1.84 x 10^{-05}	5.34 x 10^{-05}	-0.34	-1.88 x 10^{-05}	6.58 x 10^{-05}	-0.29	-1.81 x 10^{-05}	5.79 x 10^{-05}	-0.31	7.37 x 10^{-05}	6.91 x 10^{-05}	0.11
0.85	-1.84 x 10^{-05}	5.19 x 10^{-05}	-0.35	-1.93 x 10^{-05}	6.75 x 10^{-05}	-0.29	-2.00 x 10^{-05}	5.91 x 10^{-05}	-0.34	6.16 x 10^{-05}	7.10 x 10^{-05}	0.09
0.88	-1.84 x 10^{-05}	5.09 x 10^{-05}	-0.36	-1.99 x 10^{-05}	6.92 x 10^{-05}	-0.29	-2.19 x 10^{-05}	6.03 x 10^{-05}	-0.36	4.95 x 10^{-05}	7.28 x 10^{-05}	0.07
0.90	-1.85 x 10^{-05}	5.03 x 10^{-05}	-0.37	-2.04 x 10^{-05}	7.09 x 10^{-05}	-0.29	-2.37 x 10^{-05}	6.16 x 10^{-05}	-0.39	3.73 x 10^{-05}	7.46 x 10^{-05}	0.05
0.93	-1.85 x 10^{-05}	5.00 x 10^{-05}	-0.37	-2.09 x 10^{-05}	7.26 x 10^{-05}	-0.29	-2.56 x 10^{-05}	6.28 x 10^{-05}	-0.41	2.52 x 10^{-05}	7.65 x 10^{-05}	0.03
0.95	-1.85 x 10^{-05}	5.01 x 10^{-05}	-0.37	-2.14 x 10^{-05}	7.44 x 10^{-05}	-0.29	-2.76 x 10^{-05}	6.40 x 10^{-05}	-0.43	1.31 x 10^{-05}	7.83 x 10^{-05}	0.02
0.98	-1.86 x 10^{-05}	5.06 x 10^{-05}	-0.37	-2.19 x 10^{-05}	7.61 x 10^{-05}	-0.29	-2.93 x 10^{-05}	6.52 x 10^{-05}	-0.45	1.01 x 10^{-05}	8.02 x 10^{-05}	0.00
1.00	-1.86 x 10^{-05}	5.15 x 10^{-05}	-0.36	-2.24 x 10^{-05}	7.78 x 10^{-05}	-0.29	-3.12 x 10^{-05}	6.65 x 10-05	-0.47	-1.11 x 10-06	8.20 x 10^{-05}	-0.01
1.03	-1.86 x 10^{-05}	5.28 x 10^{-05}	-0.35	-2.29 x 10^{-05}	7.95 x 10^{-05}	-0.29	-3.31 x 10^{-05}	6.77 x 10-05	-0.49	-2.32 x 10-06	8.38 x 10^{-05}	-0.03
1.05	-1.87 x 10^{-05}	5.45 x 10^{-05}	-0.34	-2.34 x 10^{-05}	8.12 x 10^{-05}	-0.29	-3.49 x 10^{-05}	6.89 x 10-05	-0.51	-3.53 x 10-06	8.57 x 10^{-05}	-0.04
1.08	-1.87 x 10^{-05}	5.66 x 10^{-05}	-0.33	-2.40 x 10^{-05}	8.30 x 10^{-05}	-0.29	-3.68 x 10^{-05}	7.02 x 10-05	-0.52	-4.74 x 10-06	8.75 x 10^{-05}	-0.05
1.10	-1.87 x 10^{-05}	5.90 x 10^{-05}	-0.32	-2.45 x 10^{-05}	8.47 x 10^{-05}	-0.29	-3.87 x 10^{-05}	7.14 x 10-05	-0.54	-5.95 x 10-06	8.93 x 10^{-05}	-0.07
1.13	-1.88 x 10^{-05}	6.19 x 10^{-05}	-0.30	-2.50 x 10^{-05}	8.64 x 10^{-05}	-0.29	-4.05 x 10^{-05}	7.26 x 10-05	-0.56	-7.16 x 10-06	9.12 x 10^{-05}	-0.08
1.15	-1.88 x 10^{-05}	6.51 x 10^{-05}	-0.29	-2.55 x 10^{-05}	8.81 x 10^{-05}	-0.29	-4.24 x 10^{-05}	7.38 x 10-05	-0.57	-8.38 x 10-06	9.30 x 10^{-05}	-0.09
1.18	-1.88 x 10^{-05}	6.87 x 10^{-05}	-0.27	-2.60 x 10^{-05}	8.98 x 10^{-05}	-0.29	-4.43 x 10^{-05}	7.51 x 10-05	-0.59	-9.59 x 10-06	9.48 x 10^{-05}	-0.10
1.20	-1.89 x 10^{-05}	7.27 x 10^{-05}	-0.26	-2.65 x 10^{-05}	9.15 x 10^{-05}	-0.29	-4.61 x 10^{-05}	7.63 x 10-05	-0.60	-1.08 x 10-05	9.67 x 10^{-05}	-0.11

Table 7. Continuation...

ρ	***D'INTI***	***U(95%)***	***En***	***D'NIST***	***U(95%)***	***En***	***D'NCRC***	***U(95%)***	***En***	***D'CENAMEP***	***U(95%)***	***En***
g/cm^3	**g/cm^3**	**g/cm^3**		**g/cm^3**	**g/cm^3**		**g/cm^3**	**g/cm^3**		**g/cm^3**	**g/cm^3**	
0.80	-4.64×10^{-05}	6.77×10^{-05}	-0.69	-1.03×10^{-05}	5.41×10^{-05}	-0.19	-3.02×10^{-05}	9.35×10^{-05}	-0.32	4.24×10^{-05}	5.62×10^{-05}	0.75
0.83	-4.88×10^{-05}	6.91×10^{-05}	-0.71	-1.05×10^{-05}	5.54×10^{-05}	-0.19	-3.39×10^{-05}	9.58×10^{-05}	-0.35	3.99×10^{-05}	5.73×10^{-05}	0.70
0.85	-5.11×10^{-05}	7.06×10^{-05}	-0.72	-1.07×10^{-05}	5.67×10^{-05}	-0.19	-3.77×10^{-05}	9.80×10^{-05}	-0.38	3.74×10^{-05}	5.84×10^{-05}	0.64
0.88	-5.35×10^{-05}	7.21×10^{-05}	-0.74	-1.10×10^{-05}	5.81×10^{-05}	-0.19	-4.14×10^{-05}	1.00×10^{-04}	-0.41	3.49×10^{-05}	5.95×10^{-05}	0.59
0.90	-5.59×10^{-05}	7.36×10^{-05}	-0.76	-1.12×10^{-05}	5.94×10^{-05}	-0.19	-4.52×10^{-05}	1.03×10^{-04}	-0.44	3.24×10^{-05}	6.06×10^{-05}	0.53
0.93	-5.82×10^{-05}	7.51×10^{-05}	-0.78	-1.15×10^{-05}	6.07×10^{-05}	-0.19	-4.89×10^{-05}	1.05×10^{-04}	-0.47	2.99×10^{-05}	6.18×10^{-05}	0.48
0.95	-6.06×10^{-05}	7.66×10^{-05}	-0.79	-1.17×10^{-05}	6.21×10^{-05}	-0.19	-5.27×10^{-05}	1.07×10^{-04}	-0.49	2.74×10^{-05}	6.29×10^{-05}	0.44
0.98	-6.30×10^{-05}	7.81×10^{-05}	-0.81	-1.19×10^{-05}	6.34×10^{-05}	-0.19	-5.64×10^{-05}	1.09×10^{-04}	-0.52	2.49×10^{-05}	6.40×10^{-05}	0.39
1.00	-6.53×10^{-05}	7.96×10^{-05}	-0.82	-1.22×10^{-05}	6.47×10^{-05}	-0.19	-6.02×10^{-05}	1.11×10^{-04}	-0.54	2.23×10^{-05}	6.51×10^{-05}	0.34
1.03	-6.77×10^{-05}	8.11×10^{-05}	-0.83	-1.24×10^{-05}	6.61×10^{-05}	-0.19	-6.39×10^{-05}	1.14×10^{-04}	-0.56	1.98×10^{-05}	6.62×10^{-05}	0.30
1.05	-7.01×10^{-05}	8.26×10^{-05}	-0.85	-1.26×10^{-05}	6.74×10^{-05}	-0.19	-6.77×10^{-05}	1.16×10^{-04}	-0.58	1.73×10^{-05}	6.74×10^{-05}	0.26
1.08	-7.24×10^{-05}	8.41×10^{-05}	-0.86	-1.29×10^{-05}	6.87×10^{-05}	-0.19	-7.14×10^{-05}	1.18×10^{-04}	-0.60	1.48×10^{-05}	6.85×10^{-05}	0.22
1.10	-7.48×10^{-05}	8.56×10^{-05}	-0.87	-1.31×10^{-05}	7.01×10^{-05}	-0.19	-7.51×10^{-05}	1.20×10^{-04}	-0.62	1.23×10^{-05}	6.96×10^{-05}	0.18
1.13	-7.72×10^{-05}	8.71×10^{-05}	-0.89	-1.34×10^{-05}	7.14×10^{-05}	-0.19	-7.89×10^{-05}	1.23×10^{-04}	-0.64	9.84×10^{-06}	7.07×10^{-05}	0.14
1.15	-7.95×10^{-05}	8.85×10^{-05}	-0.90	-1.36×10^{-05}	7.27×10^{-05}	-0.19	-8.26×10^{-05}	1.25×10^{-04}	-0.66	7.34×10^{-06}	7.18×10^{-05}	0.10
1.18	-8.19×10^{-05}	9.00×10^{-05}	-0.91	-1.38×10^{-05}	7.41×10^{-05}	-0.19	-8.64×10^{-05}	1.27×10^{-04}	-0.68	4.84×10^{-06}	7.29×10^{-05}	0.07
1.20	-8.42×10^{-05}	9.15×10^{-05}	-0.92	-1.41×10^{-05}	7.54×10^{-05}	-0.19	-9.01×10^{-05}	1.29×10^{-04}	-0.70	2.34×10^{-06}	7.41×10^{-05}	0.03

Source: Authors.

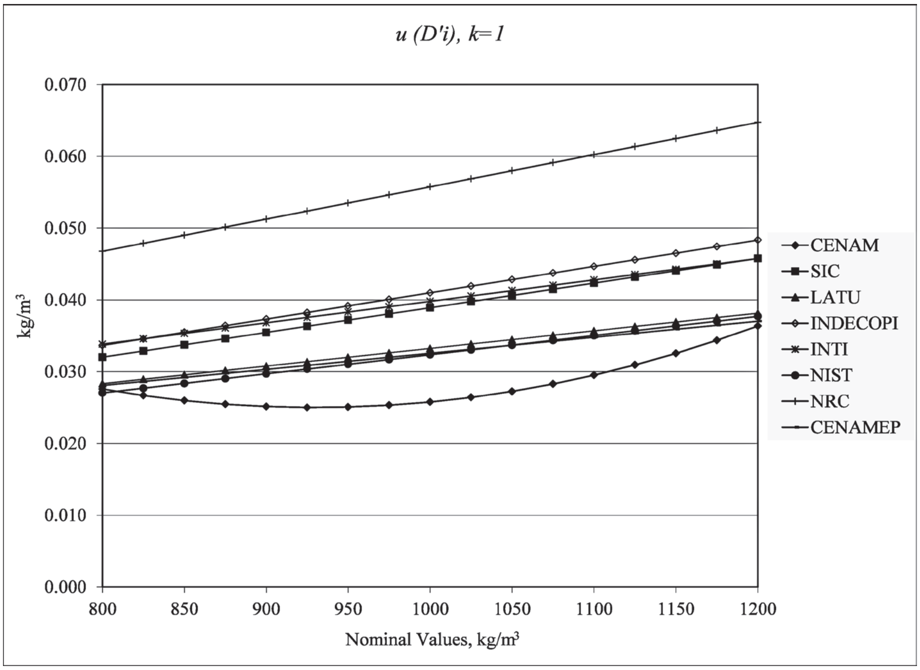

Figure 3. Standard uncertainty of *DoEs* between SIM NMIs and $KCRV_{EU}$.

Source: Authors

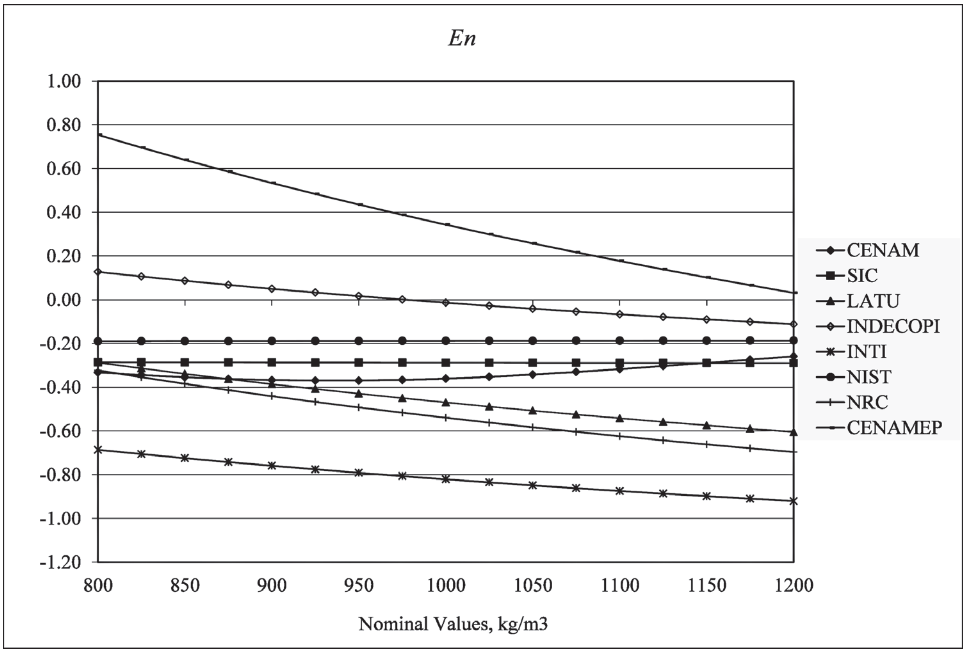

Figure 4. Normalized errors between SIM NMIs and $KCRV_{EU}$.

Source: Authors

NOTES

1. EURAMET: Europen Association of National Metrology Institutes.
2. SIM: Sistema Interamericano de Metrología.
3. The dominium of each function by the nominal values measured in the corresponding comparison.

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